2-4 Transformations of Absolute Value Functions

TEKS FOCUS

TEKS (6)(C) Analyze the effect on the graphs of $f(x) = |x|$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (1)(C), (2)(A)

VOCABULARY

- **Compression** – A compression is a transformation that decreases the distance between corresponding points of a graph and a line.
- **General form of the absolute value function** – a function of the form $f(x) = a|x - h| + k$
- **Reflection** – A reflection is a transformation that flips a graph across a line, such as the x- or y-axis.
- **Stretch** – A stretch is a transformation that increases the distance between corresponding points of a graph and a line.
- **Transformation** – A transformation of a function is a simple change to the equation of the function that results in a change in the graph of the function such as a translation or reflection.
- **Translation** – A translation is a transformation that shifts a graph vertically, horizontally, or both without changing its shape or orientation.

- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

You can quickly graph absolute value functions by transforming the graph of $y = |x|$.

take note Concept Summary Absolute Value Function Family

Parent Function $f(x) = |x|$

Translation

$y = \|x\| + d$		$y = \|x - c\|$	
$d > 0$	shifts up $\|d\|$ units	$c > 0$	shifts to the right $\|c\|$ units
$d < 0$	shifts down $\|d\|$ units	$c < 0$	shifts to the left $\|c\|$ units

Stretch, Compression, and Reflection

$y = a\|x\|$		$y = \|bx\|$	
$\|a\| > 1$	vertical stretch	$\|b\| > 1$	horizontal compression (shrink)
$0 < \|a\| < 1$	vertical compression (shrink)	$0 < \|b\| < 1$	horizontal stretch
$a < 0$	reflection across x-axis	$b < 0$	reflection across y-axis

take note

Key Concept General Form of the Absolute Value Function

When a function has a vertex, the letters h and k are used to represent the coordinates of the vertex. Because an absolute value function has a vertex, the general form is $y = a|x - h| + k$. The vertical stretch or compression factor is $|a|$, the vertex is located at (h, k), and the axis of symmetry is the line $x = h$.

Problem 1

TEKS Process Standard (1)(E)

Analyzing the Graph of $f(x) + d$ When $f(x) = |x|$

What are the graphs of the absolute value functions $y = |x| - 4$ and $y = |x| + 1$? How are these graphs different from the parent function $f(x) = |x|$?

Make a table of values that you can use to compare the y-values of each transformed function to the y-values of the parent function.

x	$\|x\|$	$\|x\| - 4$	$\|x\| + 1$
−3	3	−1	4
−1	1	−3	2
0	0	−4	1
1	1	−3	2
3	3	−1	4

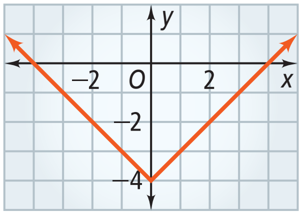
Think

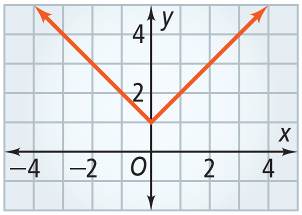
How are you changing the *y*-coordinates to get the new graphs?

For $y = |x| - 4$, you are subtracting 4 from each y-coordinate of the graph of $y = |x|$, so the graph will move downward. For $y = |x| + 1$, you are adding 1, so the graph will move up.

To draw the graph of $y = |x| - 4$, you can move the entire graph of $y = |x|$ down 4 units without changing its shape. This is a shift or translation down 4 units. The vertex is now at $(0, -4)$ instead of $(0, 0)$.

For the graph of $y = |x| + 1$, you can translate the graph of $y = |x|$ up 1 unit. The vertex of this graph is at $(0, 1)$. The axis of symmetry is still the same in both graphs.





Problem 2

Analyzing the Graph of $f(x - c)$ When $f(x) = |x|$

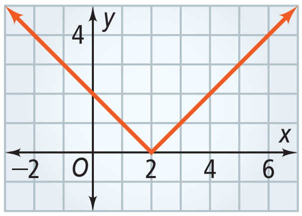
Plan

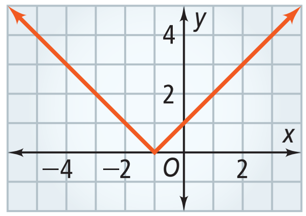
In Problem 1 you saw that the graph of $y = |x| + d$ translated the graph of $y = |x|$ up or down, depending on whether d was positive or negative. Make a table to see what adding or subtracting a number from x inside the absolute value does to the graph of the function.

What are the graphs of the absolute value functions $y = |x - 2|$ and $y = |x + 1|$? How are these graphs different from the parent function $f(x) = |x|$?

From the information in the table, you can see that the vertex of $y = |x - 2|$ is at the point (2, 0). This means you can draw the graph of $y = |x - 2|$ by translating the graph of $y = |x|$ right 2 units. Similarly, if you translate the graph of $y = |x|$ left 1 unit, you produce the graph of $y = |x + 1|$.

x	$\|x\|$	$\|x - 2\|$	$\|x + 1\|$
–3	3	5	2
–2	2	4	1
–1	1	3	0
0	0	2	1
1	1	1	2
2	2	0	3
3	3	1	4
4	4	2	5





Problem 3

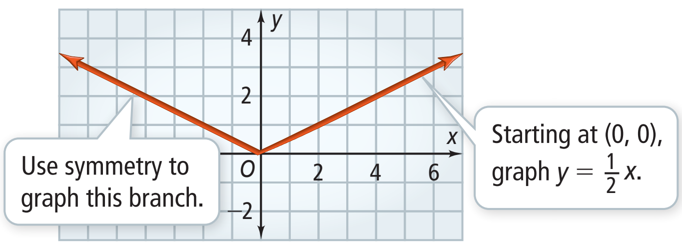
Analyzing the Graph of $af(x)$ When $f(x) = |x|$

Think

How can you describe the effect on the graph of $f(x) = |x|$? Use the equation to help you. The y-coordinate will be $\frac{1}{2}$ of what it was before.

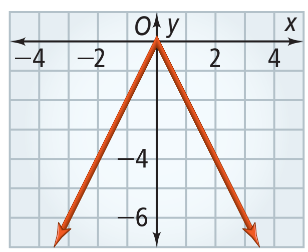
A **What is the graph of $y = \frac{1}{2}|x|$?**

The graph is a vertical compression of the graph of $f(x) = |x|$ by the factor $\frac{1}{2}$. Graph the right branch and use symmetry to graph the left branch.



B **What is the graph of $y = -2|x|$?**

Because the value of a is negative, the graph is reflected across the x-axis. Then the graph is a reflection of the graph of $f(x) = |x|$ followed by a vertical stretch by the factor 2.



Problem 4

TEKS Process Standard (1)(C)

Analyzing the Graph of $f(bx)$ When $f(x) = |x|$

Consider how different values of b affect the graph of the function $f(bx)$ when $f(x) = |x|$. Use these values for b: 1, 4, $\frac{1}{4}$, and -4.

Think

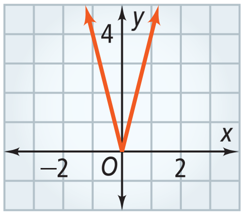
Which answer choices will help you consider graphs?
You can use paper and pencil or a graphing calculator to graph functions.

A Which tool would you use to analyze how changes in the value of b affect the graph of the function: real objects, manipulatives, paper and pencil, or technology? Why?

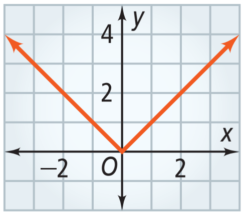
Use technology. With a graphing calculator you can graph all of the functions at once to quickly analyze how the different values of b affect the graph.

B Describe the effect on the graph by changing the value of b.

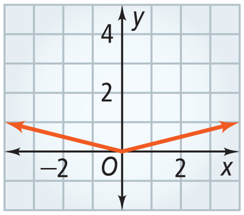
When $|b| > 1$, the graph of $f(x) = |x|$ is compressed horizontally to produce the graph of $f(x) = |bx|$.
When $0 < |b| < 1$, the graph of $f(x) = |x|$ is stretched horizontally to produce the graph of $f(x) = |bx|$.
The sign of b does not affect the graph, since the absolute value of any expression is always nonnegative.



$f(x) = |4x|$ or $f(x) = |-4x|$



$f(x) = |x|$



$f(x) = \left|\frac{1}{4}x\right|$

Problem 5

Identifying Transformations

Without graphing, what are the vertex, axis of symmetry, maximum or minimum, and x- and y-intercepts of the graph of $y = 3|x - 2| + 4$? How is the parent function $y = |x|$ transformed?

Plan

To what should you compare $y = 3|x - 2| + 4$?
Compare it to the general form, $y = a|x - h| + k$.

Compare $y = 3|x - 2| + 4$ with the general form $y = a|x - h| + k$.

$a = 3$, $h = 2$, and $k = 4$.

The vertex is (2, 4) and the axis of symmetry is $x = 2$. Because the value of a is positive, the y-coordinate of the vertex is the minimum point of the graph. The minimum is 4.

To find the x-intercept(s), set $y = 0$.

$$0 = 3|x - 2| + 4$$

$$-4 = 3|x - 2|$$

The equation $0 = 3|x - 2| + 4$ has no solutions, so there are no x-intercepts.

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Problem 5 *continued*

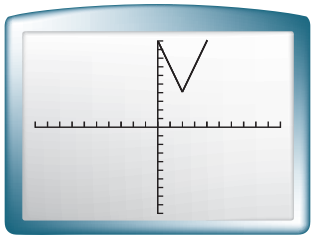
To find the y-intercept, set $x = 0$.

$$y = 3|0 - 2| + 4$$

$$y = 10$$

The y-intercept is (0, 10) or 10.

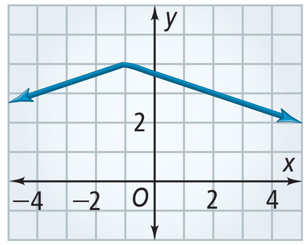
The parent function $y = |x|$ is translated 2 units to the right, vertically stretched by the factor 3, and translated 4 units up.



Check Check by graphing the equation on a graphing calculator.

Problem 6

Writing an Absolute Value Function



What is the equation of the absolute value function?

Step 1 Identify the vertex.

The vertex is at $(-1, 4)$, so $h = -1$ and $k = 4$.

Think

What does the graph tell you about *a*?
The upside-down V suggests that $a < 0$.

Step 2 Identify a.

The slope of the branch to the right of the vertex is $-\frac{1}{3}$, so $a = -\frac{1}{3}$.

Step 3 Write the equation.

Substitute the values of a, h, and k into the general form $y = a|x - h| + k$.
The equation that describes the graph is $y = -\frac{1}{3}|x + 1| + 4$.

PRACTICE and APPLICATION EXERCISES

Scan page for a Virtual Nerd™ tutorial video.

For additional support when completing your homework, go to **PearsonTEXAS.com**.

Make a table of values for each equation. Then graph the equation. Analyze the effect on the graph of the parent function $f(x) = |x|$.

1. $y = |x| + 1$ **2.** $y = |x| - 1$ **3.** $y = |x| - \frac{3}{2}$

4. $y = |x + 2|$ **5.** $y = |x + 4|$ **6.** $y = |x - 2.5|$

7. $y = |x - 1| + 3$ **8.** $y = |x + 6| - 1$ **9.** $y = |x - 3.5| + 1.5$

Graph each equation. Then analyze the effect on the graph of the parent function $f(x) = |x|$.

10. $y = 3|x|$ **11.** $y = -\frac{1}{2}|x|$ **12.** $y = -2|x|$

13. $y = \frac{1}{3}|x|$ **14.** $y = \frac{3}{2}|x|$ **15.** $y = -\frac{3}{4}|x|$

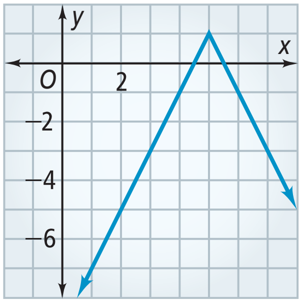
Without graphing, identify the vertex, axis of symmetry, and transformations from the parent function $f(x) = |x|$.

16. $y = |x + 2| - 4$ **17.** $y = \frac{3}{2}|x - 6|$ **18.** $y = 3|x + 6|$

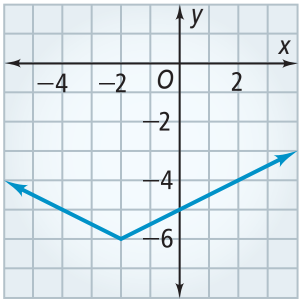
19. $y = 4 - |x + 2|$ **20.** $y = -|x - 5|$ **21.** $y = |x - 2| - 6$

Write an absolute value equation for each graph.

22.



23.

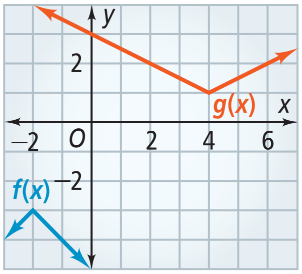


24. **Display Mathematical Ideas (1)(G)** Graph $y = -2|x + 3| + 4$. List the x- and y-intercepts, if any.

25. Graph $y = 4|x - 3| + 1$. List the vertex and the x- and y-intercepts, if any.

26. A classmate says that the graphs of $y = -3|x|$ and $y = |-3x|$ are identical. Graph each function and explain why your classmate is not correct.

27. The graphs of the absolute value functions $f(x)$ and $g(x)$ are given.



a. Describe a series of transformations that you can use to transform $f(x)$ into $g(x)$.

b. **Explain Mathematical Ideas (1)(G)** If you change the order of the transformations you found in part(a), could you still transform $f(x)$ into $g(x)$? Explain.

28. Graph each pair of equations on the same coordinate grid.

a. $y = 2|x + 1|$; $y = |2x + 1|$

b. $y = 5|x - 2|$; $y = |5x - 2|$

c. **Explain Mathematical Ideas (1)(G)** Explain why each pair of graphs in parts (a) and (b) are different.

Select Tools to Solve Problems (1)(C) **Use paper and pencil or a graphing calculator to compare the given graph to the parent function $f(x) = |x|$.**

29. $f(x) = |-x|$ **30.** $f(x) = \left|\frac{5}{2}x\right|$ **31.** $f(x) = |0.01x|$

32. Consider how different values of b affect the graph of the function $f(bx)$ when $f(x) = |x|$. What aspects of the parent function do not change for any value of b? What aspects change for particular values of b?

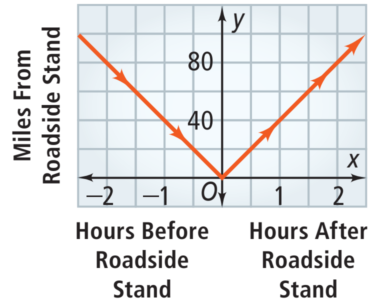
Graph each absolute value equation. Analyze the effect on the graph of the parent function $f(x) = |x|$.

33. $y = \left|-\frac{1}{4}x - 1\right|$ **34.** $y = \left|\frac{5}{2}x - 2\right|$ **35.** $y = \left|\frac{3}{2}x + 2\right|$

36. $y = |3x - 6| + 1$ **37.** $y = -|x - 3|$ **38.** $y = |2x + 6|$

39. $y = 2|x + 2| - 3$ **40.** $y = 6 - |3x|$ **41.** $y = 6 - |3x + 1|$

42. The graph at the right models the distance between a roadside stand and a car traveling at a constant speed. The x-axis represents time and the y-axis represents distance. Which equation best represents the relation shown in the graph?



A. $y = |60x|$ **C.** $y = |x| + 60$

B. $y = |40x|$ **D.** $y = |x| + 40$

43. a. Graph the equations $f(x) = -\frac{1}{2}|x - 3|$ and $g(x) = \left|-\frac{1}{2}(x - 3)\right|$ on the same set of axes.

b. **Analyze Mathematical Relationships (1)(F)** Describe the similarities and differences in the graphs.

44. a. Use a graphing calculator. Graph $y_1 = k|x|$ and $y_2 = |kx|$ for some positive value of k.

b. Graph $y_1 = k|x|$ and $y_2 = |kx|$ for some negative value of k.

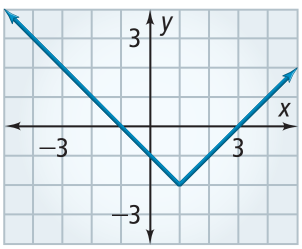
c. What conclusion can you make about the graphs of $y_1 = k|x|$ and $y_2 = |kx|$?

Graph each absolute value equation.

45. $y = |3x| - \frac{x}{3}$ **46.** $y = \frac{1}{2}|x| + 4|x - 1|$ **47.** $y = |2x| + |x - 4|$

TEXAS Test Practice

48. The graph at the right shows which equation?



A. $y = |3x - 1| + 2$ **C.** $y = |x - 1| - 2$

B. $y = |x - 1| + 2$ **D.** $y = |3x - 3| - 2$

49. How are the graphs of $y = 2x$ and $y = 2x + 2$ related?

F. The graph of $y = 2x + 2$ is the graph of $y = 2x$ translated down two units.

G. The graph of $y = 2x + 2$ is the graph of $y = 2x$ translated up two units.

H. The graph of $y = 2x + 2$ is the graph of $y = 2x$ translated to the left two units.

J. The graph of $y = 2x + 2$ is the graph of $y = 2x$ translated to the right two units.

50. What is the equation of a line parallel to $y = x$ that passes through the point (0, 1)?

A. $y = x + 1$ **B.** $y = 2x + 2$ **C.** $y = x - 1$ **D.** $y = -x$

51. Is $|y| = x$ a function? Explain.